

# BPS D-branes from an Unstable D-brane <sup>1</sup>

Chanju Kim\*, Yoonbai Kim<sup>†</sup>, Hwang-hyun Kwon<sup>†</sup>, O-Kab Kwon<sup>†</sup> and  
Chong Oh Lee\*\*

*\*Department of Physics, Ewha Womans University, Seoul 120-750, Korea*

*<sup>†</sup>BK21 Physics Research Division and Institute of Basic Science,  
Sungkyunkwan University, Suwon 440-746, Korea*

*\*\*Department of Physics, Chonbuk National University, Chonju 561-756, Korea*

## Abstract.

We search for exact tachyon kink solutions of DBI type effective action describing an unstable D-brane with worldvolume gauge field turned in both the flat and a curved background. There are various kinds of solutions in the presence of electromagnetic fields in the flat space, such as periodic arrays, topological tachyon kinks, half kinks, and bounces. We identify a BPS object,  $D(p-1)F1$  bound state, which describes a thick brane with string flux density. The curved background of interest is the ten-dimensional lift of the Salam-Sezgin vacuum and, in the asymptotic limit, it approaches  $R^{1,4} \times T^2 \times S^3$ . The solutions in the curved background are identified as composites of lower-dimensional D-branes and fundamental strings, and, in the BPS limit, they become a  $D4D2F1$  composite wrapped on  $R^{1,2} \times T^2$  where  $T^2$  is inside  $S^3$ .

**Keywords:** Unstable D-brane, Tachyon condensation, kink

**PACS:** 11.25.Uv, 11.27.+d

## INTRODUCTION AND SUMMARY

Study of the dynamics of unstable D-brane has been realized by condensation of tachyonic mode living on the brane [1]. After A. Sen wrote down the boundary state describing decay of unstable D-branes in boundary conformal field theory (BCFT) [2] and suggested the corresponding tachyon effective action [3], such observations enables us to study the decay of the unstable D-branes in the spatially homogeneous background, say, the rolling tachyon. On the other hand, spatial inhomogeneity has been another important issue, particularly in the form of tachyon solitons which form BPS and non-BPS D-branes of various codimensions.

It is well known that the tachyon effective field theory (EFT) correctly captures some aspects of the tachyon condensation in the low energy limit of open string theory. Particularly, tachyon kink solutions are effectively described in EFT. The purpose of this note is to review the tachyon kink solutions with electromagnetic fields under a runaway tachyon potential,  $1/\cosh(T/\sqrt{2})$ , in flat and a curved background, and summarizes the resulting BPS objects on the unstable D-branes.

In flat space, the obtained static kink configurations for pure tachyon fields are either singular solutions [4] or an array of regular kink-anti-kink [5, 6, 7]. Once constant DBI-type electromagnetic fields are turned on, there are

additional five non-trivial regular solutions. Specifically the solutions include two types of topological kinks, bounce, half kink, and hybrid of two half kinks [7]. When the pure electric field along the inhomogeneous direction is less than or equal to 1, corresponding BCFT solutions are also obtained in Ref. [8]. Remarkably, in the critical limit of the electric field,  $|\mathbf{E}| = 1$ , the resulting solution represents a  $D(p-1)F1$  bound state and is identified as the thick BPS-brane with string flux density. The thickness can be adjusted by the strength of the string flux density.

We attempt to extend the analysis of unstable D-brane to the case of a curved bulk background and find tachyonic kink solutions [9, 10]. Similar problems were considered in Ref. [11]. The background of our consideration is  $R^{1,4} \times T^2 \times S^3$  with non-vanishing NSNS  $B$ -field, which is the asymptotic limit of the ten-dimensional embedding [12] of the supersymmetric vacuum,  $R^{1,3} \times S^2$ , of the Salam-Sezgin model [13]. We obtain exact tachyon kink solutions on a non-BPS  $D5$ -brane whose worldvolume lies on  $R^{1,2} \times S^3$ . The obtained solutions describe the codimension-one branes on the non-BPS D-brane. In the thin limit of the solution, it becomes a BPS object with string flux density when a constant magnetic field  $h$  is goes to zero and forms a  $D4D2F1$  bound state.

## GENERALITIES

In this paper we consider the DBI-type effective action for tachyon field which couples to abelian gauge field on

<sup>1</sup> Proceeding of PASCOS 2005, Gyeongju, Korea, May 30-June 4, 2005. Talk was given by O-K. Kwon.

an unstable Dp-brane in general backgrounds,

$$S = \int d^{p+1}x \mathcal{L} = -\mathcal{T}_p \int d^{p+1}x e^{-\Phi} V(T) \sqrt{-X}, \quad (1)$$

with

$$\begin{aligned} X &\equiv \det X_{\mu\nu} = \det(g_{\mu\nu} + \partial_\mu T \partial_\nu T + \mathcal{F}_{\mu\nu}), \\ \mathcal{F}_{\mu\nu} &= B_{\mu\nu} + F_{\mu\nu}, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (\mu, \nu = 0, 1, \dots, p), \end{aligned}$$

where  $\mathcal{T}_p$  is the tension of the unstable Dp-brane,  $\Phi$  is the dilaton,  $B_{\mu\nu}$  is the induced anti-symmetric tensor. We neglected the transverse scalars which are irrelevant in our discussion. Equations of motion for the tachyon  $T$  and gauge field  $A_\mu$  are given by

$$\begin{aligned} \nabla_\mu \left[ \frac{\gamma_p}{\sqrt{-g}} C_S^{\mu\nu} \partial_\nu T \right] + \frac{\sqrt{-X}}{\sqrt{-g}} \mathcal{T}_p e^{-\Phi} \frac{dV}{dT} &= 0, \\ \nabla_\mu \left[ \frac{\gamma_p}{\sqrt{-g}} C_A^{\mu\nu} \right] &= 0, \end{aligned} \quad (2)$$

where  $C_S^{\mu\nu}$  and  $C_A^{\mu\nu}$  are the symmetric and anti-symmetric parts of the cofactor,  $C^{\mu\nu}$ , of the matrix  $(X)_{\mu\nu}$ , and we define

$$\gamma_p \equiv \frac{\mathcal{T}_p e^{-\Phi} V}{\sqrt{-X}}. \quad (4)$$

Conservation of energy-momentum is given by

$$\nabla_\mu T^{\mu\nu} = \nabla_\mu \left[ \frac{\gamma_p}{\sqrt{-g}} C_S^{\mu\nu} \right] = 0, \quad (5)$$

where  $T^{\mu\nu}$  is the energy-momentum tensor. For the tachyon potential  $V(T)$  that is symmetric under  $T \rightarrow -T$  in type IIA and IIB superstring theories, any runaway potential with  $V(0) = 1$  and  $V(\pm\infty) = 0$  is allowed for the existence of D-brane configuration of our interest, that is consistent with universal behavior in tachyon condensation [4]. Here we assume a specific form in order to obtain exact solutions

$$V(T) = \frac{1}{\cosh\left(\frac{T}{\sqrt{2}}\right)}, \quad (6)$$

which was derived in the open string theory using perturbation around half S-brane [14].

## TACHYON KINKS IN FLAT SPACE

In this section we analyze the DBI-type effective action (1) with vanishing dilaton  $\Phi$  and anti-symmetric field  $B_{\mu\nu}$  in flat  $(p+1)$ -dimensions ( $g_{\mu\nu} = \eta_{\mu\nu}$ ), and classify

all possible static tachyon kinks (See Ref. [7]). To describe the tachyon kink solutions we assume that the tachyon and gauge field strengths depend only on one spatial direction,  $x$ ,

$$T = T(x), \quad F_{\mu\nu} = F_{\mu\nu}(x). \quad (7)$$

Then the effective action (1) is simplified as

$$S = -\mathcal{T}_p \int d^{p+1}x V(T) \sqrt{\beta_p - \alpha_{px} T'^2}, \quad (8)$$

where  $T' = \partial_x T$ ,  $\alpha_{px}$  is 11-component of the cofactor  $C^{\mu\nu}$ , and  $\beta_p = -\det(\eta_{\mu\nu} + F_{\mu\nu})$ .

Bianchi identity for the abelian gauge field strength,  $dF = 0$ , the gauge field equation (3), and conservation of energy-momentum (5) determine the system completely. As expected the resulting solutions are consistent with the tachyon equation (2). What we obtain is as follows. All components of the gauge field strength are constants, and  $T^{11}$  which is written as  $T^{11} = \gamma_p \alpha_{px}$  is a constant of motion. Now the remaining equation is constancy of  $T^{11}$  where the expression of  $T^{11}$  is reexpressed by a first-order differential equation for the tachyon field,

$$\mathcal{E}_p = \frac{1}{2} T'^2 + U_p(T), \quad (9)$$

where  $\mathcal{E}_p = \beta_p/2\alpha_{px}$  and  $U_p = \alpha_{px} [\mathcal{T}_p V(T)]^2/2(T^{11})^2$ . We can classify all kink solutions in terms of three parameters ( $\alpha_{px}$ ,  $\beta_p$ ,  $T^{11}$ ). For the tachyon potential (6) all the codimension-one tachyon solitons are given by exact solutions [7].

When  $\alpha_{px}$  is negative,  $U_p$  is turned upside down and has the minimum value,  $\alpha_{px} \mathcal{T}_p^2/2(T^{11})^2$ , at  $T = 0$  and the maximum value, 0, at  $T = \pm\infty$  due to the runaway property of the tachyon potential (6). Three types of the exact solutions of Eq. (9) are

$$\sinh\left(\frac{T(x)}{\sqrt{2}}\right) = \quad (10)$$

$$\begin{cases} \sqrt{u^2 - 1} \sin(x/\zeta) & \text{for } 0 < \beta_p < \tilde{u}^2 & \text{(i)} \\ ux/\zeta & \text{for } \beta_p = 0 & \text{(ii)} \\ \sqrt{u^2 + 1} \sinh(x/\zeta) & \text{for } \beta_p < 0 & \text{(iii)} \end{cases},$$

where

$$u^2 = \left| \frac{\mathcal{T}_p^2 \alpha_{px}^2}{\beta_p (T^{11})^2} \right|, \quad \tilde{u}^2 = \beta_p u^2, \quad \zeta = \sqrt{\left| \frac{2\alpha_{px}}{\beta_p} \right|}.$$

(i) is an array of kink-anti-kink which is interpreted as an array of  $D(p-1)\bar{D}(p-1)$  ( and  $D(p-1)F1 - \bar{D}(p-1)F1$ ), and is a unique nontrivial solution in the pure tachyon case [5]. (ii) and (iii) are the topological kinks connecting two vacua  $T = \pm\infty$ , and interpreted as a single  $D(p-1)F1$

bound state. For (i) and (ii), BCFT calculation confirms this interpretation [8].

When  $\alpha_{px}$  is positive, there are three more types of non-trivial solutions

$$\sinh\left(\frac{T(x)}{\sqrt{2}}\right) = \begin{cases} \sqrt{u^2-1} \cosh(x/\zeta) & \text{for } 0 < \beta_p < \tilde{u}^2 & \text{(iv)} \\ \exp(x/\zeta) & \text{for } \beta_p = \tilde{u}^2 & \text{(v)} \\ \sqrt{1-u^2} \sinh(x/\zeta) & \text{for } \beta_p > \tilde{u}^2 & \text{(vi)} \end{cases}. \quad (11)$$

These solutions are interpreted as bounce for (iv), half-kink for (v), and hybrid of two half-kinks for (vi). The functional form of these three solutions in Eq. (11) coincides with the exact rolling tachyon solution in DBI-type tachyon effective action [15]. However, these solutions are not yet obtained in other descriptions of the system theories, such as BCFT and boundary string field theory (BSFT).

In relation to BPS nature, the single topological kink (ii) saturates BPS-type bound with thickness for  $T^{11} = \Pi^1 \neq 0$  case, i.e., the energy of this object consists of the string charge and RR-charge of the lower-dimensional D( $p-1$ )-brane exactly.

## TACHYON KINKS IN A CURVED BACKGROUND

Since the DBI action can describe the low energy dynamics of D $p$ -brane in a curved background as well, we consider the DBI-type tachyon effective action on the curved background [9, 10, 11]. This section is based on the Ref. [10], and we will study tachyon kink solution in the large dilaton limit of the ten-dimensional lift of Salam-Sezgin vacuum on  $R^{1,3} \times T^2 \times R_\rho \times S^3$  described by [12]

$$\begin{aligned} ds^2 &= dx_6^2 + 4R^2 d\rho^2 + du^2 \\ &\quad + \sin^2\left(\frac{u}{R}\right) dv^2 + \left[dw + \cos\left(\frac{u}{R}\right) dv\right]^2, \\ B &= -\cos\left(\frac{u}{R}\right) dv \wedge dw, \\ \Phi &= -\rho, \end{aligned} \quad (12)$$

where  $R$  is the radius of  $S^3$  which is parametrized by three coordinates  $(u, v, w)$ ,  $B$  is the non-vanishing NS-NS two-form field on  $S^3$ , and  $\Phi$  is the dilaton field. It is interesting to notice that the local geometry of the background (12) is nothing but the NS5-brane near horizon geometry. However, there is a difference in that string coupling constant goes to zero in the asymptotic limit of the background, while it blows up in the throat region of the NS5-brane. Thus in this background (12), it is valid to study non-BPS D-branes in terms of DBI-type effective

theory (1). Note that  $\mathcal{F}_{\mu\nu}$  defined by  $\mathcal{F}_{\mu\nu} = F_{\mu\nu} + B_{\mu\nu}$  is gauge-invariant on the worldvolume of D-brane.

We consider an unstable D5-brane on  $R^2 \times S^3$  with the coordinates  $(x, z, u, v, w)$  where  $(x, z)$  is two of the spatial coordinates of  $R^{1,3}$ . Similar to the case of the flat space in the previous section, we assume the same tachyon potential (6). The compactification scale  $R$  in the Eq. (12) is assumed to be identical to the self-dual radius  $\sqrt{2}$  in the tachyon potential (6).

From now on, we will study static solutions of the tachyon effective action (1) under the ansatz

$$\begin{aligned} T &= T(u, w), \\ F_{0z} &= E_z(u, w), \quad F_{vz} = \alpha(u, w), \quad F_{vw} = h(u, w), \end{aligned} \quad (13)$$

where we assume other components of the gauge field strength  $F_{\mu\nu}$  vanish. Using the Bianchi identity for the two-form field on the unstable D-brane,  $d\mathcal{F} = H$ , where  $H$  is field strength of the anti-symmetric tensor field, we obtain

$$E_z = \text{constant}, \quad \alpha = \text{constant}, \quad h = h(w). \quad (14)$$

Inserting the Eqs. (12), (13), and (14) into the equations of motion (2) – (3), we find that the tachyon field  $T$  is a function of either  $u$  or  $w$  but not both, and  $h$  should be a constant, i.e.,

$$\begin{aligned} \partial_u T \partial_w T &= 0, \\ h &= \text{constant}, \end{aligned} \quad (15)$$

and the dilaton field  $\Phi$  is decoupled.

We first consider the case that  $T$  depends only on  $u$ . Then we can find an exact solution

$$\sinh\left(\frac{T(u)}{\sqrt{2}}\right) = \pm \sqrt{\left(\frac{\mathcal{T}_5}{\alpha\gamma_5}\right)^2 - 1} \cos\left(\frac{u}{\sqrt{2}}\right), \quad (16)$$

where  $\gamma_5$  was defined in Eq. (4). From the analysis of the physical quantities for the solution (16), e.g., energy-momentum tensor  $T^{\mu\nu}$  and electric flux density  $\Pi^I$ , we notice that the solution represents a dimensionally reduced configuration. From the background metric in Eq. (12), the configuration spans  $T^2$  in the three-sphere:

$$\begin{aligned} ds_{S^3} &= du^2 + \sin^2\left(\frac{u}{\sqrt{2}}\right) dv^2 + \left[dw + \cos\left(\frac{u}{\sqrt{2}}\right) dv\right]^2 \\ &\xrightarrow{\frac{u}{\sqrt{2}} = \frac{\pi}{2}} ds_{T^2} = dv^2 + dw^2. \end{aligned} \quad (17)$$

To identify the RR-charge of the resulting lower dimensional D-brane we take into account the Wess-Zumino term for the unstable D-brane [4, 16],

$$S_{\text{WZ}} = \mathcal{T}_5 \int V(T) dT \wedge C_{\text{RR}} \wedge e^{F+B}. \quad (18)$$

For the solution (16) in the thin limit  $\gamma_5 \rightarrow 0$ , the Wess-Zumino term (18) is reduced to

$$S_{WZ} = \mp \pi \sqrt{2} \mathcal{T}_5 \int [C_{(5)} + \alpha C_{(3)} \wedge dz \wedge dv + h \tilde{C}_{(3)} \wedge dv \wedge dw], \quad (19)$$

where we omitted the terms containing a RR-form wedged to  $dt$ , which are irrelevant in the interpretation of the lower-dimensional D-brane. Thus the resulting configuration consists of a D4-brane stretched along  $R^2 \times T^2$  with coordinates  $(x, z, u, w)$  and RR-charge  $\mathcal{T}_4 = \sqrt{2} \pi \mathcal{T}_5$  and two D2-branes with charges per unit area,  $\sqrt{2} \pi \mathcal{T}_5 \alpha$  and  $\sqrt{2} \pi \mathcal{T}_5 h$ , spanned by the worldvolume coordinates  $(t, x, w)$  and  $(t, x, z)$ , respectively. In addition, there are fundamental strings with flux on cylinder  $R \times S_1$  of  $(z, w)$ .

In order to study the BPS nature of the solution, we now investigate the energy-momentum tensor. For the solution (16) to describe a BPS object, it is required that the pressure in  $u$ -direction,  $T^{uu}$ , and the off-diagonal stress component between two D2-branes,  $T^{zw}$ , should vanish, which are given by

$$\begin{aligned} T^{uu} &= -\frac{\gamma_5 \alpha^2}{\sin(u/\sqrt{2})}, \\ T^{zw} &= \frac{\alpha}{\sin(u/\sqrt{2})} \left[ h - \cos\left(\frac{u}{\sqrt{2}}\right) \right] \Sigma(u) \end{aligned} \quad (20)$$

with

$$\Sigma(u) = \frac{\mathcal{T}_5^2 / \gamma_5 \alpha^2}{[(\frac{\mathcal{T}_5}{\gamma_5 \alpha})^2 - 1] \cos^2(\frac{u}{\sqrt{2}}) + 1}. \quad (21)$$

To obtain this vanishing condition we should take the thin limit,  $\gamma_5 \rightarrow 0$ , with  $\pi/\sqrt{2}$  and  $h = 0$ . Then we have single D2-brane along the direction  $(x, w)$  and an electric flux along the  $z$ -direction. This solution is expected to saturate a BPS bound. The energy per unit area of coordinates  $(x, w)$  takes the form

$$\begin{aligned} \frac{\mathcal{H}}{\int dx dw} &= \int dz du dv \Pi_z + \sqrt{2} \pi \mathcal{T}_5 \alpha \int dz dv \\ &= Q_{F1} + Q_{D2}, \end{aligned} \quad (22)$$

where  $Q_{F1}$  is the total charge of fundamental strings along  $z$ -direction and  $Q_{D2}$  the total charge of D2-brane stretched along  $(x, w)$ -direction on the area  $\int dz dw$ .

In summary, when  $h = 0$ , the solution produces a BPS D4D2F1-composite in the thin limit,  $\gamma_5 \rightarrow 0$ . It consists of the D4-brane wrapped on  $R^2 \times T^2$ , the tubular D2-brane with the coordinates  $(x, w)$ , and the fundamental strings stretched along  $z$ -direction.

For the case  $T = T(w)$  we also obtain an exact solution similar to the previous solution (16) and investigate again

the BPS nature in the thin limit  $\gamma_5 \rightarrow 0$  which satisfies the condition  $h = 0$  automatically in this case. This BPS object is proven to describe the same configuration as the BPS solution in the case of  $T = T(u)$ .

## ACKNOWLEDGMENTS

This work was supported by Korea Science and Engineering Foundation (KOSEF R01-2003-000-10229-0 for C.K. and R01-2004-000-10526-0 for C.L.), and is the result of research activities (Astrophysical Research Center for the Structure and Evolution of the Cosmos (ARC-SEC)) supported by Korea Science & Engineering Foundation (Y.K., H.K., and O.K.).

## REFERENCES

1. For a review, see A. Sen, arXiv:hep-th/0410103, and references therein.
2. A. Sen, JHEP **0204**, 048 (2002) [arXiv:hep-th/0203211]; JHEP **0207**, 065 (2002) [arXiv:hep-th/0203265].
3. A. Sen, Mod. Phys. Lett. A **17**, 1797 (2002) [arXiv:hep-th/0204143].
4. A. Sen, Phys. Rev. D **68**, 066008 (2003) [arXiv:hep-th/0303057].
5. N. Lambert, H. Liu and J. Maldacena, arXiv:hep-th/0303139.
6. P. Brax, J. Mourad and D. A. Steer, Phys. Lett. B **575**, 115 (2003) [arXiv:hep-th/0304197]; E. J. Copeland, P. M. Saffin and D. A. Steer, Phys. Rev. D **68**, 065013 (2003) [arXiv:hep-th/0306294]; D. Bazeia, R. Menezes and J. G. Ramos, Mod. Phys. Lett. A **20**, 467 (2005) [arXiv:hep-th/0401195].
7. C. Kim, Y. Kim and C. O. Lee, JHEP **0305**, 020 (2003) [arXiv:hep-th/0304180]; C. Kim, Y. Kim, O. K. Kwon and C. O. Lee, JHEP **0311**, 034 (2003) [arXiv:hep-th/0305092].
8. A. Sen, Phys. Rev. D **68**, 106003 (2003) [arXiv:hep-th/0305011].
9. C. Kim, Y. Kim and O. K. Kwon, JHEP **0405**, 020 (2004) [arXiv:hep-th/0404163].
10. C. Kim, Y. Kim, H. h. Kwon and O. K. Kwon, Phys. Rev. D **72**, 026006 (2005) [arXiv:hep-th/0504130].
11. J. Kluson, JHEP **0503**, 032 (2005) [arXiv:hep-th/0411014]; JHEP **0503**, 044 (2005) [arXiv:hep-th/0501010]; arXiv:hep-th/0506250; arXiv:hep-th/0508239.
12. M. Cvetič, G. W. Gibbons and C. N. Pope, Nucl. Phys. B **677**, 164 (2004) [arXiv:hep-th/0308026].
13. A. Salam and E. Sezgin, Phys. Lett. B **147**, 47 (1984).
14. D. Kutasov and V. Niarchos, Nucl. Phys. B **666**, 56 (2003) [arXiv:hep-th/0304045].
15. C. Kim, H. B. Kim, Y. Kim and O. K. Kwon, JHEP **0303**, 008 (2003) [arXiv:hep-th/0301076].
16. K. Okuyama, JHEP **0305**, 005 (2003) [arXiv:hep-th/0304108].